Mini-course on Extreme Value Theory
January 2007
Course Description

Instructor: Professor Laurens de Haan, Erasmus University Rotterdam.

Outline: Analysis of heavy tails is important in finance, and extreme value theory is a standard statistical technique to study heavy tails. This mini-course gives an introduction to extreme value theory in the one-dimensional and multi-dimensional settings. The topics may be of interest to finance researchers studying heavy tails and formal modeling or estimation of extreme events in financial markets, such as peso problems, correlated defaults and financial crashes. The course will be focused on developing the theory for analyzing extreme events, although there will be a short application to empirical estimation using financial data.

Prerequisites: Attendees should have completed graduate courses in quantitative methods, such as mathematical modeling, econometrics or financial asset pricing.

Text:
De Haan, L. and A. Ferreira, 2006. *Extreme Value Theory: An Introduction*. Springer Press.

EXTREME VALUE THEORY

Five sessions of 2 hours

Day one and day two

- general introduction
- sketch of one-dimensional set-up
- basic theory in $\mathbb{R}^1$ (section 1.1 and some of 1.2)
- estimation of γ (Ch.3 and some of Ch.2)
- extreme quantile and tail estimation (Ch.4 and some of Ch.5)

Day three and day four

- sketch of finite-dimensional set-up
- basic theory in $\mathbb{R}^d$
- estimation of the dependence structure (Ch.7)
- estimation of probability of failure set (Ch. 8)

Day five

- basic theory in C[0,1] (Ch.9) if time permits

Note: There may be some overflow between days 2/3 and 4/5. In order to go on to $\mathbb{R}^d$, $d > 1$, one needs insight in the one-dimensional situation.

Exercises (solutions will be provided during the course)

- section 1.1: 1.2, 1.4 and 1.10. Moreover:
  - suppose that $F_1 \in \mathcal{D}(G_\gamma)$ and F_1 has right end point $x^\star$. Let F_2 be a distribution function with

    $$\lim_{t \uparrow x^\star} \frac{1 - F_2(t)}{1 - F_1(t)} = c \in (0, \infty).$$

    Prove $F_2 \in \mathcal{D}(G_\gamma)$.
- section 1.2: 1.5, 1.6, 1.7, 1.8 and 1.10. Recalculate figures 3.7 and 3.8.
- Ch.3: 3.2, 3.4 and 3.11.
- Ch.4:
  - Suppose that we already know γ in Theorem 4.4.7 so that we do not have to estimate γ . What is then the limit distribution of $\hat{p}_n$?
  - Check the relationship between 4.3.3 and fig. 4.3. How do $\hat{\gamma}$, $\hat{a}$ and $\hat{b}$ show in the picture? Choose a p_n and find $\hat{x}_n$.
  - Recalculate fig. 4.9
- Ch.6: 6.1, 6.6, 6.11 and 6.12.
- Ch.7: 7.2 and 7.6
- Ch. 7 & 8: I shall provide a two-dimensional financial data set and you will be asked to estimate the Q-function (fig.7.2), the spectral measure (fig.7.4) and the dependence coefficient e.g. $\hat{L}(1,1)$ of page 260. Further we may define a failure set and estimate its probability $p_n = P(C_n)$ (sections 8.1 and 8.2.2).